Deterministic Approach for Evaluation of Correlation Guidance Signature Quality

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The performance of a correlation guidance system is fundamentally dependent on the uniqueness quality of the measured signature. A numerical technique is described in which the probability Distribution of Signature Match Locations (DSML) is computed directly in a single computer run for a given reference signature and for given statistical properties of the measurement error. The properties of this DSML function are those of probability density functions and make it appropriate for direct use in evaluation of false fix probability as well as bounds on other error parameters such as CEP and standard deviation. Application of the technique to the problem of evaluating maps for use in terrain profile matching guidance is given.

Introduction

IN recent years, several autonomous correlation guidance concepts have been investigated for possible use in updating a navigation system on a moving platform. These concepts are based on observations of a ground-referenced signature whose unique characteristic implies the position of the platform. Typically the features of terrain that are used as signature include measurements of terrain elevation, magnetic anomaly, vertical deflection, or electromagnetic radiation. If knowledge of their spatial dependence is available prior to the observation, and if the signature is appropriate, the position of the platform may be inferred from the observation. The signature may take many forms, including those which are observed passively (e.g., an optical image) as well as those which are observed with an active transmission (e.g., a radar return from a discernible object).

The accuracy of such correlation concepts in determining the position of the platform is fundamentally related to the quality of the intrinsic signature relative to the uncertainity in observation: in simple terms, the effective "signal-to-noise ratio" for the observation. However, the complex nature of the signature (signal) has made the problem of performance prediction an interesting subject for many investigations. Some analytical results have appeared in recent publications.[1-4] However, much of the work in this area has been presented with other data that preclude its open publication. A usual approach involves limited Monte Carlo simulations as a keystone for acquiring confidence of system performance capability. In lieu of such simulations, however, two distinct analytical approaches have evolved. These are characterized with either a *statistical model for the signature*[1,2] or a *deterministic model for the signature.*[3,4] With most signatures exhibiting a random-like appearance, the desire to characterize broad classes of signature by a few parameters has motivated the former approach. However, since correlation performance is so drastically influenced by signature, it is usually of interest to consider a *specific signature form.* In such a case, although the signature may look like a random process, it is known prior to observation, and therefore can be considered in a deterministic functional relationship. The important point is that the performance that may be anticipated for a given signature (i.e., over a given area) is related to the detailed character of the signature for *that* area. Through the use of linearization applied to the signature function at the measurement points, a relationship between the "primary" correlation performance and signature gradient characteristics can be obtained.[4] However, with such an approach it must be reckoned that correlation errors are implicitly assumed to be normally distributed and thus any tendency of a given signature toward "ambiguous" or "false" fixing is not portrayed.

In previous investigations, questions of system accuracy and false fix susceptibility have required separate analyses and involve tenuous rationale to distinguish a "false fix event" from a "normal system error." In particular, one approach[1] involves the consideration of widely spread points so that the product correlation function is independent for adjoining points. In this way, only one possible fix location within the map is a correct fix and the others are implicitly false fix locations. It is recognized in this present paper, however, that both CEP and false fix behavior as well as other statistical indicators of performance are related to an *error distribution function* that is a fundamental indication of performance achievable for a given signature. From such a distribution, a directly useful measure of false fix behavior can be computed by numerically integrating the error distribution function over the appropriate central region. The resulting computation indicates the probability that error is bounded by specified limits.

The basis for the DSML function, which is an error distribution bounding the performance of an optimal estimator, lies with the application of Bayesian theory. As the name suggests, motivation for its development comes from considering a measured signature and determining the distribution of possible locations from which the signature might have originated.

Correlation Guidance Model

A correlation guidance device may be modeled as a device that makes a set of discrete signature observations, each of which is correlated to its translational and/or angular position at the instant of observation. This set of observations is thus represented as a vector function of the form:

$$z_{\text{measured}} = h(x) + w \tag{1a}$$

where z_{measured} is a vector whose elements are the discrete observations, $h(x)$ describes the dependence of the signature on the position and/or orientation of the platform, and w is a composite measurement error vector.

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The operation performed by the correlation guidance device is one of determining an estimate, $\hat{x}$, which either minimizes the difference or maximizes the correlation between the *measured vector* and a *modeled map reference vector*,

$$z_{\text{modeled}} = h(\hat{x}) \tag{1b}$$

In either case, the essential task is to determine a "best fit" in some sense between the observed signature, z_{measured}, and a modeled map reference signature, z_{modeled}.

DSML for Evaluation of Performance

The correlation guidance system performance achieved on any given mission is primarily dependent on two random processes. The first involves the random navigator errors prior to the signature observations. These errors will generally define which portion of the map is actually observed. The second process involves the signature measurement errors. These errors stem from the basic uncertainty in prediction and measurement of the signature itself and can usually be considered as independent of the prior navigation errors. The DSML concept is motivated in part by recognizing that correlation performance can be viewed as a "convolution" of these random processes. With this viewpoint, the effects of measurement errors can be addressed independently from navigation errors. In particular, the effect of measurement errors on the ability to determine the sensor position is evaluated, given that the sensor has been placed at some unknown location and that measurement errors are statistically described. Having characterized this "ability" (which is limited by signature "signal-to-noise" considerations) in the form of a probability density function, other locations are considered and the corresponding results are weighted according to a statistical description of a priori expected locations.

Let the particular set of signature measurements for a given mission be expressed in the form,

$$z_{j\ell} = h(x_j) + w_\ell \tag{2}$$

where the subscript j denotes the position (navigator error dependent) when the observation is made and ℓ denotes the particular sample of an ensemble of possible measurement error sets.

Given the measurement set $z_{j\ell}$, the ability to distinguish between the various x_k is now evaluated by application of Bayes rule:

$$Pr(x_k/z_{j\ell}) = \frac{p(z_{j\ell}/x_k)Pr(x_k)}{p(z_{j\ell})} \tag{3}$$

In this and following discussions, $Pr(\ )$ refers to a discrete probability and $p(\ )$ refers to a probability density of the variate. $Pr(x_k/z_{j\ell})$ then refers to the discrete probability that x_k is the correct position, given a measurement signature of location x_j. The subscript ℓ denotes that this probability is dependent on a particular measurement error, w_ℓ. The term, $p(z_{j\ell}/x_k)$, is the probability density of the measurement sample, $z_{j\ell}$ given x_k, and represents the distribution of $z_{j\ell}$ about the modeled signature, $h(x_k)$.

The distribution function, $Pr(x_k/z_{j\ell})$ is an important indication of the certainty with which a decision can be made. For example, a wise policy might be to choose the maximum of the function $Pr(x_k/z_{j\ell})$. However, the uncertainty of such a choice for that particular observation, $z_{j\ell}$, is reflected in the relative magnitude of the probability associated with competing values of x_k in Eq. (3). A more useful term for *signature evaluation* is the expectation of $Pr(x_k/z_{j\ell})$ with respect to the measurement noise w_ℓ in $z_{j\ell}$. This quantity, denoted as the "conditional" Distribution of Signature Match Locations, is defined

$$\bar{P}r(x_k/x_j) = \underset{w_\ell}{E}\{Pr(x_k/z_{j\ell})\}$$

$$= \int_{z_{j\ell}} Pr(x_k/z_{j\ell})p(z_{j\ell}/x_j)\,\mathrm{d}z_{j\ell} \tag{4}$$

where $Pr(x_k/z_{j\ell})$ is defined by Eq. (3) and $E\{\ \}$ denotes the expectation with respect to the variable indicated below, i.e., w_ℓ in this case.

For estimating (prior to the mission) the performance to be expected on a mission, the expression for conditional DSML defined in Eq. (4) must now be convolved with the distribution for possible observed signatures, each referenced by x_j. Defining the error $\delta = x_k - x_j$, the resulting error distribution is obtained

$$P_{\text{DSML}}(\delta) = \sum_j \bar{P}r(x_j+\delta/x_j)Pr(x_j) \tag{5a}$$

$$= \sum_j \int_{z_{j\ell}} Pr(x_j+\delta/z_{j\ell})p(z_{j\ell}/x_j)Pr(x_j)\,\mathrm{d}z_{j\ell} \tag{5b}$$

The last two factors in the integrand may be replaced using Bayes' rule

$$P_{\text{DSML}}(\delta) = \sum_j \int_{z_{j\ell}} Pr(x_j+\delta/z_{j\ell}) \times Pr(x_j/z_{j\ell})p(z_{j\ell})\,\mathrm{d}z_{j\ell} \tag{5c}$$

which may be expressed in the form,

$$P_{\text{DSML}}(\delta) = E_{z_{j\ell}}\Big[\sum_j \{Pr(x_j+\delta/z_{j\ell})Pr(x_j/z_{j\ell})\}\Big] \tag{5d}$$

Interpretation of the DSML Fuction†

The error distribution function, $P_{\text{DSML}}(\delta)$, derived in the preceding section, may be interpreted in terms of a multihypothesis statistical decision rule. The hypotheses are that given a measured signature z, the true location of the observation platform, is one of several potential discrete values of x within a given set, $S{:}x_1, x_2, \ldots x_n$. If a decision rule, d, selectes an estimate $\hat{x}$ based on a measurement z, the probability of error, δ, is denoted by $Pe(\delta)$,

$$Pe(\delta) = E_z\Big[\sum_{\hat{x}} \mathrm{d}(\hat{x}/z)Pr(\hat{x}+\delta/z)\Big] \tag{6}$$

For example, the maximum likelihood decision rule is

$$\mathrm{d}(\hat{x}/z) = \mathrm{d}_M(\hat{x}/z) = \begin{cases} 1 & \text{if } Pr(\hat{x}/z) = \max_x\{Pr(x/z)\} \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

which gives a probability of zero error, i.e., $\delta = 0$, to be

$$Pe_M(0) = E_z[\max_x Pr(x/z)] \tag{8}$$

Referring to Eq. (5d), it may noted that the DSML distribution can be interpreted as probability of error for the decision rule

$$d(\hat{x}/z) = Pr\ (\hat{x}/z) \tag{9}$$

†An alternate development of these results may be found in Ref. 5.

which implies that the DSML decision rule is equivalent to a random number generator whose output is described by the distribution $Pr(x/z)$.

From such an interpretation, it is apparent that the probability of error as computed from the distribution $P_{DSML}(\delta)$ is a conservative estimate of error for a maximum likelihood decision rule. In fact, this conservative nature of the decision rule suggests the existence of bounds that are closely related to those derived by Devijver.[6,7] In these references, error bounds were defined in terms of a "Bayesian Distance" function in the general context of pattern recognition. For correlation guidance, these bounds are translated to be less than a specified magnitude, M. This cumulative probability is expressed by summing terms in Eq. (5d) for $|\delta| \leq M$.

$$F_{DSML}(M) \triangleq \text{Prob } (|\delta| \leq M)$$

$$= \sum_{|\delta| \leq M} E_z\{ \sum_x Pr(x/z)Pr(x+\delta/z) \} \tag{10}$$

where the subscripts are deleted for notational convenience. The bounds are simply stated: (A formal proof is included in the Appendix.)

1) $F_{DSML}(M)$ establishes an upper bound for the probability that error will be larger than M, given a decision rule,

$$d(x/z) = d_0(\hat{x}/z)$$

$$= \begin{cases} 1 \text{ if } \sum_{|\delta| \leq M} Pr(\hat{x}+\delta) = \max_x \left[\sum_{|\delta| \leq M} Pr(x+\delta) \right] \\ 0 \quad \text{otherwise} \end{cases} \tag{11}$$

i.e., one which maximizes the likelihood of $|\delta| \leq M$. Thus

$$[1 - F_{DSML}(M)] \geq [1 - F_0(M)] \tag{12a}$$

or

$$F_{DSML}(M) \leq F_0(M) \tag{12b}$$

where $F_0(M)$ is the expected cumulative error for the decision rule $d_0(\hat{x}/z)$.

2) $[F_{DSML}(2M)]^{1/2}$ establishes a lower bound for the probability that error will be larger than M, given *any* decision rule

$$1 - [F_{DSML}(M)]^{1/2} \leq 1 - F(M) \tag{13a}$$

or

$$F(M) \leq [F_{DSML}(2M)]^{1/2} \tag{13b}$$

Numerical Evaluation of $P_{DSML}(\delta)$

The formal definition of P_{DSML} presented so far is that given in Eq. (5). However, for evaluating the function it is necessary to expand this definition using Bayes' rule. Writing Eq. (5c) without subscripts, and reordering, we have

$$P_{DSML}(\delta) = \int_z \sum_x Pr(x/z)Pr(x+\delta/z)p(z)\,dz \tag{14}$$

Substituting Bayes' rule,

$$Pr(x/z) = [Pr(x)p(z/x)]/p(z) \tag{15}$$

into Eq. (14) gives,

$$P_{DSML}(\delta) = \int_z \sum_x \left[\frac{Pr(x)p(z/x)}{p(z)} \times \frac{Pr(x+\delta)p(z/x+\delta)}{p(z)} \right] P(z)\,dz \tag{16a}$$

which is

$$P_{DSML}(\delta) = \sum_x Pr(x)Pr(x+\delta) \times \int_z \frac{p(z/x)p(z/x+\delta)}{p(z)}\,dz \tag{16b}$$

The implementation of Eq. (16b) requires assumptions on the forms of the distributions to allow analytic integration over z. It is obvious that $p(z/x)$ is a shifted replica of $p(w)$ since measurement noise is assumed additive. One reasonable model for $p(w)$ is a jointly normal distribution

$$p(w) = K_1 \exp[-(w'w/2\sigma_w^2)] \tag{17}$$

where the prime is used to denote the transpose. Thus,

$$p(z/x) = K_1 \exp - \left\{ \frac{[z-h(x)]'[z-h(x)]}{2\sigma_w^2} \right\} \tag{18}$$

where $h(x)$ is the mapped signature function of Eq. (1). The product of $p(z/x)$ and $p(z/x+\delta)$ is

$$p(z/x)p(z/x+\delta) = K_2 \exp \left\{ - \frac{2z'z - 2z'[h(x)+h(x+\delta)]}{2\sigma_w^2} + \frac{h'(x)h(x) + h'(x+\delta)h(x+\delta)}{2\sigma_w^2} \right\} \tag{19}$$

where K_2 is the composite normalizing constant.

The function $p(z)$ represents the probability distribution for the signature vector z and is defined

$$p(z) = \sum_x Pr(x)p(z/x) \tag{20}$$

However, the numerical evaluation of Eq. (20) is prohibitive. A more rational approach is to assume that the elements of z are gaussian-distributed, independent random variables. Then

$$p(z) = K_3 \exp - \left\{ \frac{[z-\bar{z}]'[z-\bar{z}]}{2\sigma_z^2} \right\} \tag{21}$$

where $\bar{z}$ is the expected value of z and

$$\sigma_z^2 = \sigma_w^2 + \sigma_h^2 \tag{22}$$

where σ_h^2 is the variance of the $h(x)$ function. The integrand in Eq. (16b) is then expressed

$$\frac{p(z/x)p(z/x+\delta)}{p(z)} = K_4 \exp \left\{ -z'z \left[\frac{1}{\sigma_w^2} - \frac{1}{2\sigma_z^2} \right] + z' \left[\frac{h(x)+h(x+\delta)}{\sigma_w^2} - \frac{\bar{z}}{\sigma_z^2} \right] - \left[\frac{h(x)'h(x) + h(x+\delta)'h(x+\delta)}{2\sigma_w^2} \right] + \frac{\bar{z}'\bar{z}}{2\sigma_z^2} \right\} \tag{23}$$

By using the definite integral,[8]

$$\int_{-\infty}^{\infty} \exp(-a^2u^2 + bu)\,du = \frac{(\pi)^{1/2}}{a} \exp\left(\frac{b^2}{4a^2}\right) \tag{24}$$

the vector integration required in Eq. (16b) may be obtained in the form,

$$\int_z \frac{p(z/x)p(z/x+\delta)}{p(z)}\,dz = K_5 \exp[C(x,\delta)] \tag{25}$$

where

$$C(x,\delta) = -\frac{[h(x)-h(x+\delta)]'[h(x)-h(x+\delta)]}{4\sigma_w^{\ 2}} + \frac{[h(x)+h(x+\delta)-2\bar{z}]\ [h(x)+h(x+\delta)-2\bar{z}]}{4(\sigma_w^{\ 2}+2\,\sigma_h^2)} \tag{26}$$

The distribution $P_{\rm DSML}(\delta)$ is therefore written in the form,

$$P_{\rm DSML}(\delta) = \sum_x Pr(x)\ [K\,Pr(x+\delta)\ \exp\{C(x,\delta)\}] \tag{27}$$

where K is defined for all $x \epsilon S$, the domain of the reference map, such that summation over the "conditional" probability density functions is unity, i.e.,

$$\sum_{|\delta|} K\,Pr(x+\delta)\exp[C(x,\delta)] = 1 \tag{28}$$

The form of Eq. (26) emphasizes the effect that the noise statistics have on error probability relative to the quality of the signature characteristics $[h(x)-h(x+\delta)]'[h(x)-h(x+\delta)]$. The distribution of x required in this equation is the "discretized" navigation error distribution and may be computed by cell to cell integration of the continuous distribution, modeled in the form,

$$p(x) = K_6 \exp - \left[\frac{x'x}{2\sigma_x^2}\right] \tag{29}$$

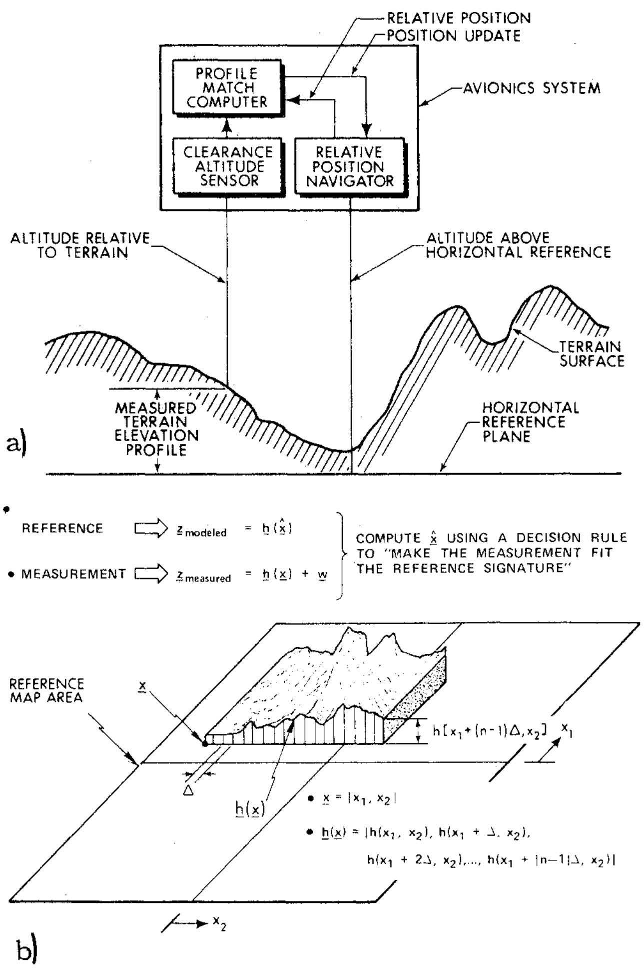

Fig. 1 Terrain elevation profile matching concept: a) measurement process; b) correlation process.

Example Results

The concept of terrain profile matching[9] is an interesting example of correlation guidance for demonstrating the DSML evaluation approach. In this concept, the measured signature is a profile of terrain elevation measurements of the form illustrated in Figs. 1a and 1b. The ensemble of possible observed signatures is stored in the form of a digitized terrain elevation map. A correlation algorithm (profile match computer) is implemented to determine the location within the map corresponding to the observed signature. This location is then used for the navigation update.

For analysis of terrain profile matching, the measured signature is represented by the vector $z_{\rm measured}$ of Eq. (1) whose elements are the measured terrain elevations at each of n locations distributed uniformly along the profile and where $h(x)$ is the profile (or vector) of elevations referenced to location x, and w is a vector whose elements are Gaussian-distributed white-noise samples.

The distribution function, $P_{\rm DSML}(\delta)$, was computed according to Eq. (27) using a digitized USGS map for an area with modest terrain features. In general, the shape of the function, $P_{\rm DSML}(\delta)$, is dependent on system parameters including profile length and the measurement noise σ_w, as well as the particular features of the map evaluated. For example, a longer profile length and/or a smaller measurement noise will lead to a more "compact" distribution. However, it should be noted that subtle terrain elevation profile similarities found within the map are also reflected in the shape of $P_{\rm DSML}(\delta)$. For example, the function illustrated in Fig. 2 typifies the case where a potential observed profile has a high correlation with a remote, "false-fix" profile.

For more detailed analysis, it is often of interest to examine certain of the "conditional" distribution functions prior to weighting with the navigator uncertainty $Pr(x)$ in forming the composite function $P_{\rm DSML}(\delta)$ defined by Eq. (26). Such functions (illustrated in Fig. 3a) indicate the quality of signature for an observation resulting from a particular value of the initial navigation error (denoted by a dot on the grid). The composite function is shown in Fig. (3b). Having computed $P_{\rm DSML}$ according to Eq. (27), the cumulative probability function is readily obtained by summing over values of this function for $|\delta| \leq M$. Although a square map limits this summation to likewise square regions, a radius of an equivalent circular area, R_0, is reasonable for association with the cumulative probability values, with

$$R_0 = (2j-1)\Delta/\sqrt{\pi}; j = 1,2,3,\ldots,n \tag{30}$$

where Δ is the spacing of the reference map. Figure 4 illustrates the resulting cumulative probability bounds com-

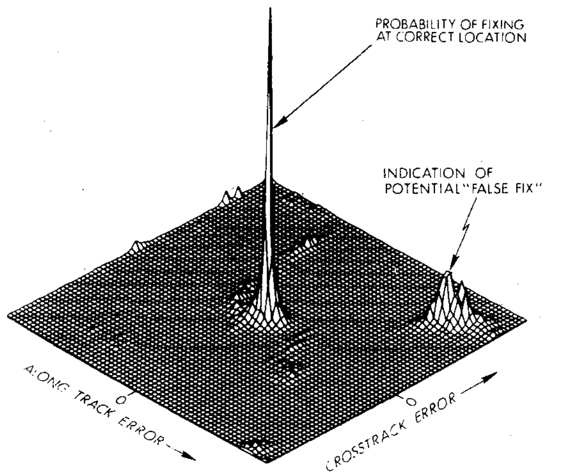

Fig. 2 Error distribution, $P_{\rm DSML}(\delta)$, for an ambiguous measurement profile.

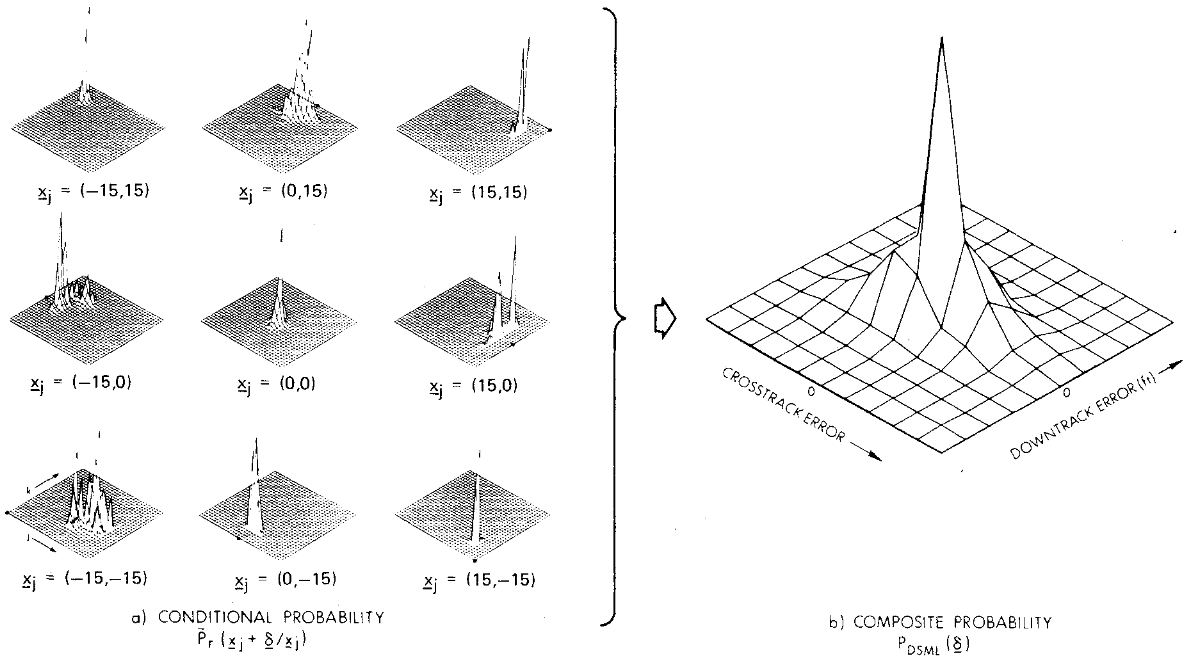

Fig. 3 Error distributions for signature evaluation.

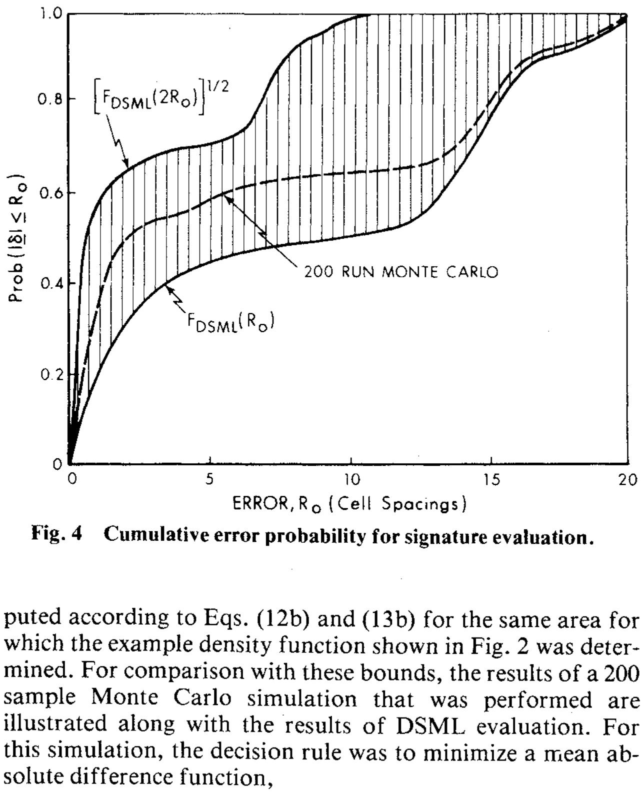

Fig. 4 Cumulative error probability for signature evaluation.

puted according to Eqs. (12b) and (13b) for the same area for which the example density function shown in Fig. 2 was determined. For comparison with these bounds, the results of a 200 sample Monte Carlo simulation that was performed are illustrated along with the results of DSML evaluation. For this simulation, the decision rule was to minimize a mean absolute difference function,

$$J(x) = \sum_j |h_j(x) - z_j| \tag{31}$$

where $h_j(x)$ and z_j are elements of the vectors $h(x)$ and z, respectively. The Monte Carlo simulation required approximately 10 times the computation time required for the DSML evaluation.

The type of results displayed in Fig. 4 are particularly appropriate for signature evaluation since they depict in a single graph, error characteristics such as CEP as well as false fix behavior. For example, from Fig. 4 we may infer an upper bound on CEP of approximately 12 units. Similarly, the probability of a fix with an error greater than 15 units is approximately 0.3.

Conclusions

The quality of signature to be used in a correlation guidance system is a primary influence on the performance of the system. A conventional method for evaluating the quality or uniqueness of a signature for a specific correlation algorithm is the Monte Carlo simulation of experimental results of the algorithm. An alternative approach involves the use of the DSML probability function developed herein to establish bounds on the performance of an optimal correlation guidance concept that uses a digitized reference map. Several advantages of this approach are apparent. First of all, evaluation is *not* dependent on any specific algorithm but rather it is related to the intrinsic quality of the actual correlation signature. The requirement for exhaustive, algorithm-dependent Monte Carlo simulations is avoided. For most problems it is anticipated that significant computational savings over Monte Carlo simulation may be realized. Finally, the signature characteristics that determine the important correlation error behavior are embodied in a concise and easily interpreted graph of error probability as a function of error magnitude. This graph quantifies system accuracy as well as false fix characteristics. Error distribution functions leading to this final graph provide additional insight regarding the nature of false fix behavior.

Appendix: Proof of DSML Bounds

1) The upper bound for the probability that correlation error, $|\delta| \leq M$ is

$$[1 - F_0(M)] \leq [1 - F_{\text{DSML}}(M)] \tag{A1a}$$

or

$$F_{\text{DSML}}(M) \leq F_0(M) \tag{A1b}$$

where $F_0(M)$ is the cumulative error probability for a decision rule, d_0, that maximizes $Pr(|\delta| \leq M)$.

Proof: By definition

$$F_0(M) = E_z\left\{\max_x \left[\sum_{|\delta| \leq M} Pr(x+\delta)/z) \right]\right\} \tag{A2}$$

$$=E_z\left\{\sum_{|\delta|\le M} Pr(\hat{x}_0+\delta/z)\right\} \quad \text{(A3)}$$

where

$$\sum_{|\delta|\le M} Pr(x_0+\delta/z)$$

$$=\max_x\left[\sum_{|\delta|\le M} Pr(x+\delta/z)\right] \quad \text{(A4)}$$

Also by definition

$$F_{\text{DSML}}(M)=E_z\left\{\sum_x \sum_{|\delta|\le M} Pr(x/z)Pr(x+\delta/z)\right\}$$

$$=E_z\left\{\sum_x Pr(x/z)\left[\sum_{|\delta|\le M} Pr(x+\delta/z)\right]\right\} \quad \text{(A5)}$$

which gives

$$F_{\text{DSML}}(M)\le E_z\left\{\sum_x Pr(x/z)\left[\sum_{|\delta|\le M} Pr(\hat{x}_0+\delta/z)\right]\right\} \quad \text{(A6)}$$

in light of Eq. (A4). But

$$\sum_x Pr(x/z)=1 \quad \text{(A7)}$$

Thus

$$F_{\text{DSML}}(M)\le E_z\left[\sum_{|\delta|<M} Pr(\hat{x}_0+\delta/z)\right]=F_0(M) \quad \text{(A8)}$$

2) The lower bound for the probability that correlation error $|\delta|\le M$ is

$$[1-F(M)]\ge 1-[F_{\text{DSML}}(2M)]^{1/2} \quad \text{A9a)}$$

or

$$[F_{\text{DSML}}(2M)]^{1/2}\ge F(M) \quad \text{(A9b)}$$

where $F(M)$ is the cummulative error probability for any decision rule.

Proof: From Eq. (A5)

$$F_{\text{DSML}}(2M)=E_z\left\{\sum_x Pr(x/z)\left[\sum_{|\delta|\le 2M} Pr(x+\delta/z)\right]\right\} \quad \text{(A10)}$$

Assume an arbitrary decision rule $\hat{d}$ that selects an estimate $\hat{x}$. Let

$$x=\hat{x}+k$$

then Eq. (A10) may be expressed

$$F_{\text{DSML}}(2M)=E_z\left\{\sum_{|k|\le M} Pr(\hat{x}+k/z)\right.$$

$$\times\sum_{|\delta|\le 2M} Pr(\hat{x}+k+\delta/z)+\sum_{|k|\ge M} Pr(\hat{x}+k/z)$$

$$\left.\times\sum_{|\delta|\le 2M} Pr(\hat{x}+k+\delta/z)\right\} \quad \text{(A11)}$$

Within the first product of summations term of Eq. (A11), there is a subset for which

$$|\delta'|=|k+\delta|\le M \quad \text{(A12)}$$

Thus we may write

$$F_{\text{DSML}}(2M)=E_z\left\{\sum_{|k|\le M} Pr(\hat{x}+k/z)\right.$$

$$\sum_{|\delta'|\le M} Pr(\hat{x}+\delta'/z)$$

$$\left.+\text{ other positive terms}\right\} \quad \text{(A13)}$$

Thus, since both summations are independent, we may write

$$F_{\text{DSML}}(2M)\ge E_z\left\{\left[\sum_{|\delta|\le M} Pr(\hat{x}+\delta/z)\right]^2\right\} \quad \text{(A14)}$$

However, since it is always true that

$$E\{y^2\}=[E\{y\}]^2+\sigma_y^2\ge[E\{y\}]^2 \quad \text{(A15)}$$

then it follows that

$$F_{\text{DSML}}(2M)\ge\left[E_z\left\{\sum_{|\delta|\le M} Pr(\hat{x}+\delta]\right\}\right]^2$$

$$=[F(M)]^2 \quad \text{(A16)}$$

or

$$[F_{\text{DSML}}(2M)]^{1/2}\ge F(M) \quad \text{(A17)}$$

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